Information Geometric Modeling of Scattering Induced Quantum Entanglement

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We present an information geometric analysis of entanglement generated by an s-wave scattering between two Gaussian wave packets. We conjecture that the pre and post-collisional quantum dynamical scenarios related to an elastic head-on collision are macroscopic manifestations emerging from microscopic statistical structures. We then describe them by uncorrelated and correlated Gaussian statistical models, respectively. This allows us to express the entanglement strength in terms of scattering potential and incident particle energies. Furthermore, we show how the entanglement duration can be related to the scattering potential and incident particle energies. Finally, we discuss the connection between entanglement and complexity of motion.

PACS numbers: Probability Theory (02.50.Cw), Riemannian Geometry (02.40.Ky), Complexity (89.70.Eg), Entropy (89.70.Cf), Quantum Entanglement (03.65.Ud).

One of the most important features of composite quantum mechanical systems is their ability to become entangled [1, 2]. In general, quantum entanglement is described by quantum correlations among the distinct subsystems of the entire composite quantum system. For such correlated quantum systems, it is not possible to specify the quantum state of any subsystem independently of the remaining subsystems [1]. Apart from these remarks, the fundamental meaning of quantum entanglement is still a widely debated issue [3].

From a conceptual point of view, the simplest and most realistic mechanism of generating entanglement between two particles is via scattering processes [4, 5]. The two particles can become entangled as they approach each other as a consequence of mutual interactions. For instance, for interaction potentials with a strong repulsive core, quantum interference between incident and reflected waves can generate transient entanglement. After the collision, the two particles may still be entangled and share forms of quantum information in the final scattered state. Quantum entanglement can also be generated during inelastic collisions between the dissipative walls of a container and the quantum system confined within it [6]. Entanglement may also be induced in multi-atom systems confined in a harmonic trap interacting via a delta interaction potential [7].

In order to obtain a clear and detailed understanding of entanglement, it is first necessary to quantify it. It is known that for pure maximally entangled states the quantum state of any subsystem is maximally mixed, while for separable states it is pure. Thus, one is led to consider the von Neumann entropy of the reduced state, measuring its degree of mixedness, as an entanglement measure. This turns out to be correct for pure bipartite states [8] (the case we are considering in this Letter), while for more general states other entanglement measures should be invoked [9, 10]. Apart from the above presented remarks, a great deal remains unclear about the physical interpretation of entanglement measures [11] and much remains unsatisfactory about our understanding of scattering-induced quantum entanglement, especially with regard to how interaction potentials and particle energies control the entanglement [4]. Finally, our knowledge of the connections between entanglement and complexity of motion remains far from complete [12].

In this Letter we investigate the potential utility of the *Information Geometric Approach to Chaos* (IGAC) [13, 14] in characterizing the quantum entanglement produced by a head-on elastic collision between two Gaussian wave packets interacting via a scattering process [15].

IGAC is a theoretical framework developed to study the complexity of informational geodesic flows on curved statistical manifolds underlying the probabilistic description of physical, biological or chemical systems. It is the information geometric analogue of conventional geometrodynamical approaches [16, 17] where the classical configuration space is replaced by a statistical manifold with the additional possibility of considering complex dynamics arising from non conformally flat metrics (the Jacobi metric is always conformally flat). For recent applications of the IGAC to quantum physics we refer to [18, 19].

Here we conjecture that the scattering induced quantum entanglement is a macroscopic manifestation emerging from specific statistical microstructures. Specifically, using information geometric techniques [20] and inductive inference methods [21, 22], we propose that the pre and post-collisional scenarios are modelled by an uncorrelated [23] and

correlated Gaussian statistical model [24], respectively. We present an analytical connection between the entanglement strength - quantified in terms of purity - to the scattering potential and incident particle energies. Furthermore, we relate the entanglement duration to the scattering potential and incident particle energies. Finally, we uncover a quantitative relation between quantum entanglement and the information geometric complexity of motion [25].

Before describing the physical system being studied, we recall that spatially localized Gaussian wave packets are especially useful to describe naturally occurring quantum states and they are easy to handle since many important properties of these states can often be obtained in an analytic fashion [26]. Furthermore, the Wigner distribution of Gaussian wave packets is positive-definite and therefore Gaussian states could be tagged as essentially classical [27].

The physical system being considered consists of two interacting spin-0 particles of equal mass m. For such a system, a complete set of commuting observables is furnished by the momentum operators of each particle [28]. In terms of the center of mass and relative coordinates, the Hamiltonian $\mathcal{H}$ of the system becomes [15],

$$\mathcal{H} = \mathcal{H}_{\text{cm}} + \mathcal{H}_{\text{rel}}, \tag{1}$$

with

$$\mathcal{H}_{\text{cm}} = \frac{P^2}{2M} \text{ and, } \mathcal{H}_{\text{rel}} = \frac{p^2}{2\mu} + V(x), \tag{2}$$

where $M \equiv 2m$ is the total mass and $\mu \equiv \frac{m}{2}$ is the reduced mass. The interaction potential $V(x)$ is isotropic and has a short range d such that $V(x) \approx 0$ for $x > d$. Before colliding, the two particles are in the form of disentangled Gaussian wave packets, each characterized by a width σ_0 in momentum space. The initial distance between the two particles is R_0 and their average initial momenta - setting the Planck constant $\hbar$ equal to one - are $\mp k_0$, respectively. We emphasize that the three-dimensional scattering process in [15] can be effectively reduced to a one-dimensional process as far as the probability density-based analysis concerns. This can be explained by observing that both the pre and post-collisional three-dimensional Gaussian wave-packets in [15] are isotropic. That is to say, in spherical coordinates the representations of the separable two-particle states exhibit a functional dependence on the radial variable only. For this reason the three-dimensional vectorial representations may be effectively reduced to one-dimensional representations. After some algebra, it follows that the initial (pre-collisional) two-particle square wave amplitude in momentum space reads,

$$P_{\text{pre}}^{(\text{QM})}(k_1, k_2|k_0, -k_0, \sigma_0) = \frac{1}{2\pi\sigma_0^2} \exp\left[-\frac{(k_1-k_0)^2 + (k_2+k_0)^2}{2\sigma_0^2}\right], \tag{3}$$

where $\pm k_0$ are the expected values of the momenta k_1 and k_2, respectively. The square root of the variance for each momentum is denoted by σ_0. Similarly, following [15] and our above mentioned observation on the dimensional reduction of the analysis, after some tedious algebra it turns out that the final (post-collisional) two-particle square wave amplitude in momentum space in the low energy s-wave scattering approximation becomes,

$$P_{\text{post}}^{(\text{QM})}(k_1, k_2|k_0, -k_0, \sigma_0; r_{\text{QM}}) = \frac{\exp\left\{-\frac{1}{2\sigma_0^2(1-r_{\text{QM}}^2)}\left[(k_1-k_0)^2 - 2r_{\text{QM}}(k_1-k_0)(k_2+k_0) + (k_2+k_0)^2\right]\right\}}{2\pi\sigma_0^2\sqrt{1-r_{\text{QM}}^2}}, \tag{4}$$

where the adimensional coefficient r_{QM} is defined as,

$$r_{\text{QM}} = r_{\text{QM}}(k_0, \sigma_0, R_0, a_S) \stackrel{\text{def}}{=} \sqrt{8(2k_0^2 + \sigma_0^2) R_0 a_s}, \tag{5}$$

where $r_{\text{QM}} \ll 1$ and a_s is the s-wave scattering length. As a side remark, we point out that (4) reduces to (3) when r_{QM} vanishes.

As pointed out earlier, in order to properly analyze entanglement, the entanglement entropy obtained from the long time limit post-collisional wave function is required. In most cases however, this must be performed numerically. Thus, to approach the problem analytically and simultaneously gain insights into the problem, it is convenient to make use of the linearized version of the entropy of the system, i.e. of the purity of the system [15]. The purity function is defined as $\mathcal{P} \stackrel{\text{def}}{=} \text{Tr}(\rho_A^2)$ where $\rho_A \equiv \text{Tr}_B(\rho_{AB})$ is the reduced density matrix of particle A and ρ_{AB} is the two-particle density matrix associated with the post-collisional two-particle wave function. For pure two-particle states, the smaller the value of $\mathcal{P}$ the higher the entanglement. That is, the loss of purity provides an indicator of the degree of entanglement. Hence, a disentangled product state corresponds to $\mathcal{P} = 1$. We emphasize that the purity has been used as a measure of the degree of entanglement in various physical situations [29], especially in atomic physics

in order to characterize the two-body correlations in dynamical atomic processes [30, 31]. Under the assumption that the two particles are well separated both initially (before collision) and finally (after collision) [32] and assuming that the colliding Gaussian wave packets are very narrow in the momentum space ($\sigma_0 \ll 1$ such that the phase shift can be treated as a constant $\theta(k_0) \equiv \theta_0$), it follows that the purity of the post-collisional two-particle wave function is approximately given by [15]

$$\mathcal{P} = 1 - \frac{4\left(2k_0^2 + \sigma_0^2\right) R_0 \sqrt{S_0}}{\sqrt{\pi}} + \mathcal{O}\left(S_0\right), \tag{6}$$

where $S_0 \overset{\text{def}}{=} 4\pi \left|f\left(k_0\right)\right|^2$ is the scattering cross section and $f\left(k_0\right) = \frac{e^{i\theta_0}\sin\theta_0}{k_0} \approx \frac{\theta_0}{k_0}$ is the s-wave scattering amplitude.

Although very important, the analysis of [15] does not address the problem of how the interaction potentials and particle energies control the scattering-induced entanglement (a key problem as pointed out in [4]) and it does not discuss any possible connection between the entanglement generated in the scattering process and the complexity of the motion related to the pre and post-collisional quantum dynamical scenarios (a key problem as pointed out in [12]).

In this Letter, we attempt to provide some answers to such unsolved relevant issues. We conjecture that the pre and post-collisional quantum dynamical scenarios characterized by (3) and (4), respectively, and describing the quantum entanglement (quantified in terms of the purity $\mathcal{P}$ in (6)) produced by a head-on collision between two Gaussian wave packets are macroscopic manifestations emerging from specific underlying microscopic statistical structures. We stress that within the IGAC, we provide a probabilist description of physical systems by studying the temporal evolution on curved statistical manifolds of probability distributions encoding all the relevant available information concerning the system considered. For the problem under investigation, we propose that $P_{\text{pre}}^{(\text{QM})}\left(k_1,\, k_2|k_0,\, -k_0,\, \sigma_0\right)$ can be interpreted as a limiting case (initial time limit) arising from an evolving uncorrelated Gaussian probability distribution $P_{\text{pre}}^{(\text{IG})}\left(k_1,\, k_2|\mu_{k_1}\left(\tau\right),\, \mu_{k_2}\left(\tau\right),\, \sigma\left(\tau\right)\right)$,

$$P_{\text{pre}}^{(\text{IG})}\left(k_1,\, k_2|\mu_{k_1}\left(\tau\right),\, \mu_{k_2}\left(\tau\right),\, \sigma\left(\tau\right)\right) \overset{\text{def}}{=} \frac{\exp\left\{-\frac{1}{2\sigma^2}\left[\left(k_1 - \mu_{k_1}\right)^2 + \left(k_2 - \mu_{k_2}\right)^2\right]\right\}}{2\pi\sigma^2}. \tag{7}$$

As a matter of fact, setting $\mu_{k_1}(0) = k_0$, $\mu_{k_2}(0) = -k_0$ and $\sigma(0) = \sigma_0$, we obtain

$$P_{\text{pre}}^{(\text{IG})}\left(k_1,\, k_2|k_0,\, -k_0,\, \sigma_0\right) = P_{\text{pre}}^{(\text{QM})}\left(k_1,\, k_2|k_0,\, -k_0,\, \sigma_0\right). \tag{8}$$

Similarly, we propose that $P_{\text{post}}^{(\text{QM})}\left(k_1,\, k_2|k_0,\, -k_0,\, \sigma_0;\, r_{\text{QM}}\right)$ can be viewed as a limiting case (final time limit) arising from a correlated Gaussian probability distribution $P_{\text{post}}^{(\text{IG})}\left(k_1,\, k_2|\mu_{k_1}\left(\tau\right),\, \mu_{k_2}\left(\tau\right),\, \sigma\left(\tau\right);\, r_{\text{IG}}\right)$,

$$P_{\text{post}}^{(\text{IG})}\left(k_1,\, k_2|\mu_{k_1}\left(\tau\right),\, \mu_{k_2}\left(\tau\right),\, \sigma\left(\tau\right);\, r_{\text{IG}}\right) = \frac{\exp\left\{-\frac{\left[\left(k_1-\mu_{k_1}\right)^2 - 2r_{\text{IG}}\left(k_1-\mu_{k_1}\right)\left(k_2-\mu_{k_2}\right) + \left(k_2-\mu_{k_2}\right)^2\right]}{2\sigma^2\left(1-r_{\text{IG}}^2\right)}\right\}}{2\pi\sigma^2\sqrt{1-r_{\text{IG}}^2}}, \tag{9}$$

where $r_{\text{IG}} \overset{\text{def}}{=} \frac{\langle k_1 k_2\rangle - \langle k_1\rangle\langle k_2\rangle}{\sigma^2}$ is the correlation coefficient. Indeed, setting $\mu_{k_1}\left(\tau_{\text{final}}\right) = k_0$, $\mu_{k_2}\left(\tau_{\text{final}}\right) = -k_0$ and $\sigma\left(\tau_{\text{final}}\right) = \sigma_0$, we obtain

$$P_{\text{post}}^{(\text{IG})}\left(k_1,\, k_2|k_0,\, -k_0,\, \sigma_0;\, r_{\text{IG}}\right) = P_{\text{post}}^{(\text{QM})}\left(k_1,\, k_2|k_0,\, -k_0,\, \sigma_0;\, r_{\text{QM}}\right) \tag{10}$$

in the limit of weak correlations with $r_{\text{IG}} \equiv r_{\text{QM}} \ll 1$.

At this stage our conjecture is only mathematically sustained by the formal identities (8) and (10). To render our conjecture also physically motivated, recall that s-wave scattering can also be described in terms of a scattering potential $V(x)$ and the scattering phase shift $\theta(k)$. For the problem under consideration, $V(x)$ equals the constant value V for $0 \le x \le d$ while it is vanishing for $x > d$. The quantities V and d denote the height (for $V > 0$; repulsive potential) or depth (for $V < 0$; attractive potential) and range of the potential, respectively. Within the standard quantum framework, integrating the radial part of Schrödinger equation with this potential for the scattered wave and imposing the matching condition at $x = d$ for its solution and its first derivative leads to [33]

$$\tan\theta_0 = \frac{k_{\text{out}}\tan\left(k_{\text{in}}d\right) - k_{\text{in}}\tan\left(k_{\text{out}}d\right)}{k_{\text{in}} + k_{\text{out}}\tan\left(k_{\text{out}}d\right)\tan\left(k_{\text{in}}d\right)}, \tag{11}$$

with $k_{\text{in}} = \sqrt{2\mu\left(T - V\right)}$ for $0 \le x \le d$, $k_{\text{out}} = \sqrt{2\mu T}$ for $x > d$ and $\theta_0 \approx -k_0 a_s$ denotes the s-wave scattering phase shift. The quantities μ and T are the reduced mass and kinetic energy of the two-particle system in the relative

coordinates, respectively; k_{in} and k_{out} represent the conjugate-coordinate wave vectors inside and outside the potential region, respectively. Equation (11) indicates that the scattering potential $V(x)$ shifts the phase of the scattered wave at points beyond the scattering region.

Within the IGAC, given the curved statistical manifolds of probability distributions in (7) and (9), we compute the Fisher-Rao information metrics on each manifold. We then integrate the two sets of geodesic equations (three equations for each set) leading to the expected trajectories connecting the initial and final macroscopic configurations defined by the macroscopic variables $\left(\mu_{k_1}(\tau), \mu_{k_2}(\tau), \sigma(\tau)\right)$. In particular, we obtain that

$$k_{\text{in}} = \sqrt{1 - r_{\text{IG}}}\, k_{\text{out}}. \tag{12}$$

Considering Eq. (12) together with the limit of low energy s-wave scattering ($k_0 d \ll 1$) and low correlations ($r_{\text{IG}} \ll 1$), the matching condition (11) in the information geometric framework reads

$$\theta(k_0) \approx -\frac{1}{3} r_{\text{IG}} d^3 k_0^3, \tag{13}$$

where

$$r_{\text{IG}} = \frac{V}{T} = \frac{2\mu V}{k_0^2}. \tag{14}$$

Combining (13) and (14), we obtain

$$\theta(k_0) \approx -\frac{2}{3} \mu V d^3 k_0. \tag{15}$$

Equation (15), obtained via information geometric dynamical methods, is in perfect agreement with the result presented in [34] (and not in [15]) where standard Schrodinger's quantum dynamics was employed. Such finding is especially important because it allows to state that our conjecture is also physically motivated.

As a consequence of (6) and (13), we find that when both low energy and weak correlation regimes occur, the purity $\mathcal{P}$ of the system becomes

$$\mathcal{P} \approx 1 - \frac{16\mu V \left(2k_0^2 + \sigma_0^2\right) R_0 d^3}{3}. \tag{16}$$

Equation (16) implies that the purity $\mathcal{P}$ can be expressed in terms of physical quantities such as the scattering potential $V(x)$ and the initial quantities k_0, σ_0 and R_0 via (14). Apart from (15), Eq. (16) is the first significant finding obtained within our hybrid approach (quantum dynamical results combined with information geometric modeling techniques) that allows to explain how the interaction potential $V(x)$ and the incident particle energies T control the strength of the entanglement. The role played by r_{IG} in the quantities $\mathcal{P}$ and V suggests that the physical information about quantum scattering and therefore about quantum entanglement is encoded in the statistical correlation coefficient, specifically in the covariance term $\text{Cov}(k_1, k_2) \overset{\text{def}}{=} \langle k_1 k_2 \rangle - \langle k_1 \rangle \langle k_2 \rangle$ appearing in the definition of r_{IG}.

Within the IGAC we are also able to estimate the statistical temporal duration over which the entanglement is active. Indeed, it turns out that the uncorrelated and the correlated statistical Gaussian models would require time intervals $\tau_{\text{uncorr.}}$ and $\tau_{\text{corr.}}$, respectively, to attain the same value as the initial momentum k_0. Assuming $r_{\text{IG}} \ll 1$, from the above-mentioned geodesic equation analysis it can be shown that what we define "*entanglement duration*" $\Delta(k_0, \sigma_0, r_{\text{IG}})$ reads,

$$\Delta(k_0, \sigma_0, r_{\text{IG}}) \overset{\text{def}}{=} \tau_{\text{corr.}} - \tau_{\text{uncorr.}} \propto \left| \ln\left\{1 - \left[(1 - r_{\text{IG}})^{-1/2} - 1\right] \cdot \eta_\Delta(k_0, \sigma_0)\right\}\right|, \tag{17}$$

where $\eta_\Delta = \eta_\Delta(k_0, \sigma_0)$ is given by

$$\eta_\Delta(k_0, \sigma_0) = \left(\frac{k_0}{\sigma_0}\right)^2 \exp\left[\left(\frac{\sigma_0}{k_0}\right)^2 - \frac{3}{4}\left(\frac{\sigma_0}{k_0}\right)^4 + \mathcal{O}\left[\left(\frac{\sigma_0}{k_0}\right)^6\right]\right] \quad \text{for} \quad \frac{\sigma_0}{k_0} \ll 1. \tag{18}$$

Here, we can find the upper bound value of r_{IG} by means of (17) and (18),

$$r_{\text{IG}} < \frac{2}{\eta_\Delta(k_0, \sigma_0)}. \tag{19}$$

For example, with $\sigma_0/k_0 \sim 10^{-3}$ we have $r_{\rm IG} < 2 \times 10^{-6}$. We observe that the entanglement duration can be controlled via the initial parameters k_0, σ_0 and the correlations $r_{\rm IG}$ (therefore via the incident particle energies and the scattering potential due to (14)). Also, we notice that in the absence of correlations, i.e. $r_{\rm IG} \to 0$, $\Delta \to 0$. It is anticipated that the maximum duration would be obtained when $r_{\rm IG}$ is the greatest and the ratio σ_0/k_0 is the smallest. In summary, the entanglement duration allows to quantitatively estimate the temporal interval over which the entanglement is active in terms of statistical geodesic paths (statistical evolution of probability distributions) on the statistical manifolds underlying the information dynamics used to describe the pre and post-collisional scenarios (absence and presence of entanglement, respectively). It encodes information about how long it would take for an entangled system to overcome the momentum gap (relative to a corresponding non-entangled system) generated by the scattering phase shift. The entangled system only attains the full value of momentum (i.e. the momentum value as seen in the corresponding non-entangled system) when the scattering phase shift vanishes. For this reason, the entanglement duration represents the statistical temporal duration over which the entanglement is active.

Our final finding uncovers an interesting quantitative connection between quantum entanglement quantified by the purity $\mathcal{P}$ in (16) and the information geometric complexity of motion on the uncorrelated and correlated curved statistical manifolds $\mathcal{M}_s^{(\rm uncorr.)}$ and $\mathcal{M}_s^{(\rm corr.)}$, respectively. The information geometric complexity (IGC) represents the volume of the effective parametric space explored by the system in its evolution between the chosen initial and final macrostates. In general, the volume itself is in general given in terms of a multidimensional fold-integral over the geodesic paths connecting the initial and final macrostates [25],

$$\mathcal{C}_{\rm IGC} \overset{\rm def}{=} \frac{1}{\tau}\int_0^\tau d\tau' vol\left[\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right)\right], \tag{20}$$

where

$$vol\left[\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right)\right] \overset{\rm def}{=} \int_{\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right)} \rho_{(\mathcal{M}_s,\, g)}\left(\theta^1,..., \theta^n\right) d^n\Theta. \tag{21}$$

The quantity $\rho_{(\mathcal{M}_s,\, g)}\left(\theta^1,..., \theta^n\right)$ is the so-called Fisher density and equals the square root of the determinant of the metric tensor $g_{\mu\nu}\left(\Theta\right)$,

$$\rho_{(\mathcal{M}_s,\, g)}\left(\theta^1,..., \theta^n\right) \overset{\rm def}{=} \sqrt{g\left(\left(\theta^1,..., \theta^n\right)\right)}. \tag{22}$$

The integration space $\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right)$ in (21) is defined as follows,

$$\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right) \overset{\rm def}{=} \left\{\Theta \equiv \left(\theta^1,..., \theta^n\right) : \theta^k\left(0\right) \le \theta^k \le \theta^k\left(\tau'\right)\right\}, \tag{23}$$

where $k = 1,.., n$ and $\theta^k \equiv \theta^k\left(s\right)$ with $0 \le s \le \tau'$ such that,

$$\frac{d^2\theta^k\left(s\right)}{ds^2} + \Gamma_{lm}^k \frac{d\theta^l}{ds}\frac{d\theta^m}{ds} = 0. \tag{24}$$

The integration space $\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right)$ in (23) is a n-dimensional subspace of the whole (permitted) parameter space $\mathcal{D}_\Theta^{(\rm tot)}$. The elements of $\mathcal{D}_\Theta^{(\rm geodesic)}\left(\tau'\right)$ are the n-dimensional macrovariables $\{\Theta\}$ whose components θ^k are bounded by specified limits of integration $\theta^k\left(0\right)$ and $\theta^k\left(\tau'\right)$ with $k = 1,.., n$. The limits of integration are obtained via integration of the n-dimensional set of coupled nonlinear second order ordinary differential equations characterizing the geodesic equations. In our case $n = 3$ with $\Theta \equiv \left(\mu_{k_1}\left(\tau\right), \mu_{k_2}\left(\tau\right), \sigma\left(\tau\right)\right)$ and following the line of reasoning presented in [23, 24] one finds that

$$\mathcal{C}_{\rm IGC}^{(\rm corr.)} = \sqrt{\frac{1 - r_{\rm IG}}{1 + r_{\rm IG}}}\mathcal{C}_{\rm IGC}^{(\rm uncorr.)}, \tag{25}$$

where $\mathcal{C}_{\rm IGC}^{(\rm corr.)}$ and $\mathcal{C}_{\rm IGC}^{(\rm uncorr.)}$ denotes the information geometric complexities of motion on the chosen statistical manifolds. As a side remark, we point out that (25) confirms that an increase in the correlational structure of the dynamical equations for the statistical variables labelling a macrostate of a system implies a reduction in the complexity of the geodesic paths on the underlying curved statistical manifolds [35, 36]. In other words, making macroscopic predictions in the presence of correlations is easier than in their absence. Combining (16) and (25) it follows that

$$\mathcal{P} \approx 1 - \eta_{\mathcal{C}}\left(k_0,\, \sigma_0\right) \cdot \frac{\Delta\mathcal{C}^2}{\mathcal{C}_{\rm total}^2}, \tag{26}$$

where,

$$\Delta\mathcal{C}^2 \stackrel{\text{def}}{=} \left[\mathcal{C}_{\text{IGC}}^{(\text{uncorr.})}\right]^2 - \left[\mathcal{C}_{\text{IGC}}^{(\text{corr.})}\right]^2,\, \mathcal{C}_{\text{total}}^2 \stackrel{\text{def}}{=} \left[\mathcal{C}_{\text{IGC}}^{(\text{uncorr.})}\right]^2 + \left[\mathcal{C}_{\text{IGC}}^{(\text{corr.})}\right]^2 \tag{27}$$

and,

$$\eta_{\mathcal{C}}\left(k_0,\, \sigma_0\right) = \frac{8}{3}k_0^2\left(2k_0^2+\sigma_0^2\right)R_0 d^3. \tag{28}$$

From (26) it is evident that the scattering-induced quantum entanglement and the information geometric complexity of motion are connected. It found that when purity goes to unity, the difference between the correlated and non-correlated information geometric complexities approaches zero. In particular, our analysis allows to conclude that the information geometric complexity of motion (viewed as an indicator of how "difficult" is to make macroscopic predictions) of Gaussian wave-packets decreases when the quantum wave-packets becomes entangled. This is reminiscent of the fact that quantum entanglement is generally considered a very useful resource at our disposal in quantum computing. As a side remark, we would like to emphasize that the quantification of the complexity of quantum motion represents a quite delicate task and it is also a very debatable issue. Indeed, there is no unique manner in which such complexity can be described. In particular, the notion of information geometric complexity employed in this Letter differs from that introduced in [12] where the number of harmonics of the Wigner function was chosen as a suitable quantum signature of complexity of motion.

In conclusion, our information geometric characterization gives a novel probabilistic picture of quantum entanglement. Within our framework, the key-feature is the study of the statistical evolution of classical probability distributions on curved statistical manifolds underlying the information dynamics used to describe the pre and post-collisional scenarios (absence and presence of entanglement, respectively). The emergence of entanglement manifests itself with a change in the probability distribution that quantifies our state of knowledge about the system. Once this is accepted (indeed we provide both mathematical and physical supports to our conjecture, see Eqs. (8), (10) and (13), respectively), our analysis becomes relevant for the following reasons: first, we are able to quantitatively express the entanglement strength, quantified by purity, in terms of scattering potential and incident particle energies (see Eq. (16)). The scattering potential and incident particle energies are in turn related to the micro-correlation coefficient r_{IG}, a quantity that parameterizes the correlated microscopic degrees of freedom of the system (see Eq. (14)). Second, we introduce a new quantity termed "entanglement duration" which characterizes the temporal duration over which the entanglement is active and show that it can be controlled by the initial momentum p_0, momentum spread σ_0 and r_{IG} (see Eq. (17)). Finally, we uncover a quantitative relation between quantum entanglement and information geometric complexity (see Eq. (26)). We also point out that our analysis allows us to interpret quantum entanglement as a perturbation of statistical space geometry in analogy to the interpretation of gravitation as perturbation of flat spacetime. The nature of the perturbation of statistical geometry is global. This, together with the time-independence of the geometry, leads to the notion of non-locality. The perturbation of statistical geometry is associated with the scattering phase shift in the momentum space.

We are confident that the present work represents significant progress toward the goal of understanding the relationship between statistical microcorrelations and quantum entanglement on the one hand and the effect of microcorrelations on the dynamical complexity of informational geodesic flows on the other. It is our hope to build upon the techniques employed in this work to ultimately establish a sound information-geometric interpretation of quantum entanglement together with its connection to complexity of motion in more general physical scenarios.

Acknowledgments

This work was supported by WCU (World Class University) program of NRF/MEST (R32-2009-000-10130-0) and by the European Community's Seventh Framework Program FP7/2007-2013 under grant agreement 213681 (CORNER Project).

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